Lecture Notes October 26, 2010

Economic General Equilibrium

General Equilibrium Theory: Who was Prof. Debreu and why did he have his own parking space in Berkeley's Central Campus??

Nobel Prizes: Arrow, Debreu

June 1993: A birthday party for mathematical general equilibrium theory!

October 2005: Mathematical Economics: The Legacy of Gerard Debreu
http://emlab.berkeley.edu/users/cshannon/debreu/home.htm

What does mathematical general equilibrium theory do? Tries to put microeconomics on same basis of logical precision as algebra or geometry. Axiomatic method: allows generalization; clearly distinguishes assumptions from conclusions and clarifies the links between them.

Four ideas about writing an economic theory:

Ockam's razor (KISS - Keep it simple, stupid.), improves generality

Testable assumptions (logical positivism), avoids vacuity

Link with experience, robustness, Solow "All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect. " (Contribution to the Theory of Economic Growth, 1956)

Precision, reliable results, Hugo Sonnenschein: "In 1954, referring to the first and second theorems of classical welfare economics, Gerard wrote 'The contents of both Theorems ... are old beliefs in economics. Arrow and Debreu have recently treated these questions with techniques permitting proofs.' This statement is precisely correct; once there were beliefs, now there was knowledge.

"But more was at stake. Great scholars change the way that we think about the world, and about what and who we are. The Arrow-Debreu model, as communicated in *Theory of Value* changed basic thinking, and it quickly became the standard model of price theory. It is the 'benchmark' model in Finance, International Trade, Public Finance, Transportation, and even macroeconomics. ... In rather short order it was no longer 'as it is' in Marshall, Hicks, and Samuelson; rather it became 'as it is' in *Theory of Value*." (remarks at the Debreu conference, Berkeley, 2005).

Partial and General Economic Equilibrium

PARTIAL EQUILIBRIUM

$S_k(p^o_k) = D_k(p^o_k)$, with $p^o_k > 0$ (or possibly, $p^o_k = 0$), or

$p^o_k = 0$ if $S_k(p^o_k) > D_k(p^o_k)$.

GENERAL EQUILIBRIUM For all i = 1,...,N,

$D_i(p^o_1, p^o_2,...,p^o_N) = S_i(p^o_1,..,p^o_N)$, $p^o_i > 0$, and

$p^o_i = 0$ for goods i such that

$D_i(p^o_1,...,p^o_N) < S_i(p^o_1,...,p^o_N)$.

What's wrong with partial equilibrium? Suppose there's no consistent choice of $(p^o_1,..,p^o_N)$. Then there would be (apparent) partial equilibrium --- viewing each market separately --- but no way to sustain it, because of cross-market interaction. Competitive equilibrium is supposed to make efficient use of resources by minimizing costs and allowing optimizing consumer choice. But how do we know prices in other markets reflect underlying scarcity assuming "other things being equal" . If not, then apparently efficient equilibrium allocation may be wasteful. A valid notion of equilibrium and efficiency needs to take cross-market interaction into account.

Three big ideas

Equilibrium: S(p) = D(p)
Decentralization
Efficiency

The Edgeworth Box

2 person, 2 good, pure exchange economy

Fixed positive quantities of X and Y, and two households, 1 and 2.

Household 1 is endowed with $\bar{X}^1$ of good X and $\bar{Y}^1$ of good Y, utility function $U^1(X^1, Y^1)$. Household 2 is endowed with $\bar{X}^2$ of good X and $\bar{Y}^2$ of good Y, utility function $U^2(X^2, Y^2)$

$X^1 + X^2 = \bar{X}^1 + \bar{X}^2 \equiv \bar{X}$,
$Y^1 + Y^2 = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y}$.

Each point in the Edgeworth box represents an attainable choice of X^1 and X^2 , Y^1 and Y^2 .
1's origin is at the southwest corner; 1's consumption increases as the allocation point moves in a northeast direction; 2's increases as the allocation point moves in a southwest direction. Superimpose indifference curves on the Edgeworth Box.

Competitive Equilibrium

(p^o_x , p^o_y) so that (X^{o1}, Y^{o1}) maximizes $U^1(X^1, Y^1)$ subject to
$(p^o_x , p^o_y)\cdot(X^1, Y^1) \leq (p^o_x , p^o_y)\cdot(\bar{X}^1, \bar{Y}^1)$ and
(X^{o2}, Y^{o2}) maximizes $U^2(X^2, Y^2)$ subject to
$(p^o_x , p^o_y)\cdot(X^1, Y^1) \leq (p^o_x , p^o_y)\cdot(\bar{X}^2, \bar{Y}^2)$, and
$(X^{o1}, Y^{o1}) + (X^{o2}, Y^{o2}) = (\bar{X}^1, \bar{Y}^1)+(\bar{X}^2, \bar{Y}^2)$
or $(X^{o1}, Y^{o1}) + (X^{o2}, Y^{o2}) \leq (\bar{X}^1, \bar{Y}^1)+ (\bar{X}^2, \bar{Y}^2)$, where the inequality holds co-ordinatewise and any good for which there is a strict inequality has a price of 0.

Pareto efficiency:

An allocation is Pareto efficient if all of the opportunities for mutually desirable reallocation have been fully used. The allocation is Pareto efficient if there is no available reallocation that can improve the utility level of one household while not reducing the utility of any household.

Tangency of 1 and 2's indifference curves : Pareto efficient allocations.

Pareto efficient allocation:
$(X^{o1} , Y^{o1}), (X^{o2}, Y^{o2})$ maximizes

$U^1(X^1 , Y^1)$ subject to

$U^2(X^2 , Y^2) \geq U^{o2}$ (typically equality will hold and $U^{o2}=U^2(X^{o2}, Y^{o2})$) and subject to the resource constraints

$$X^1 + X^2 = \bar{X}^1 + \bar{X}^2 \equiv \bar{X}$$

$$Y^1 + Y^2 = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y} \quad .$$

Equivalently, $X^2 = \bar{X} - X^1$, $Y^2 = \bar{Y} - Y^1$

<u>Solving for Pareto efficiency</u> (Assuming differentiability and an interior solution):

Lagrangian

$$L \equiv U^1(X^1, Y^1) + \lambda[U^2(\bar{X} - X^1, \bar{Y} - Y^1) - U^{o2}]$$

$$\frac{\partial L}{\partial X^1} = \frac{\partial U^1}{\partial X^1} - \lambda \frac{\partial U^2}{\partial X^2} = 0$$

$$\frac{\partial L}{\partial Y^1} = \frac{\partial U^1}{\partial Y^1} - \lambda \frac{\partial U^2}{\partial Y^2} = 0$$

$$\frac{\partial L}{\partial \lambda} = U^2(X^2, Y^2) - U^{o2} = 0$$

This gives us then the condition

$$MRS^1_{xy} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = MRS^2_{xy}$$ or equivalently

$$MRS^1_{xy} = \frac{\partial Y^1}{\partial X^1}\Big|_{U^1=\text{constant}} = \frac{\partial Y^2}{\partial X^2}\Big|_{U^2=\text{constant}} = MRS^2_{xy}$$

Pareto efficient allocation in the Edgeworth box: the slope of 2's indifference curve at an efficient allocation will equal the slope of 1's indifference curve; the points of tangency of the two curves.

contract curve = individually rational Pareto efficient points

<u>Market allocation</u>

p^x, p^y

Household 1:Choose X^1, Y^1, to maximize $U^1(X^1,Y^1)$ subject to

$p^xX^1 + p^yY^1 = p^x \bar{X}^1 + p^y\bar{Y}^1 = B^1$

budget constraint is a straight line passing through the endowment point $(\bar{X}^1, \bar{Y}^1)$ with slope $-\frac{p^x}{p^y}$.

Lagrangian

$L = U^1(X^1,Y^1) - \lambda\, [p^x X^1 + p^y Y^1 - B^1]$

$$\frac{\partial L}{\partial X} = \frac{\partial U^1}{\partial X^1} - \lambda p^x = 0$$

$$\frac{\partial L}{\partial Y} = \frac{\partial U^1}{\partial Y^1} - \lambda p^y = 0$$

Therefore, at the utility optimum subject to budget constraint we have

$$MRS^1_{xy} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{p^x}{p^y}$$; Similarly for household 2,

$$MRS^2_{xy} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \frac{p^x}{p^y}\quad .$$

Equilibrium prices: p^{*x} and p^{*y} so that

$$X^{*1} + X^{*2} = \bar{X}^1 + \bar{X}^2 \equiv \bar{X}$$
$$Y^{*1} + Y^{*2} = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y}\quad ,$$

(market clearing)

where X^{*i} and Y^{*i} , i =1, 2, are utility maximizing mix of X and Y at prices p^{*x} and p^{*y} .

$$-\frac{\partial Y^1}{\partial X^1}\Big|_{U^1=U^{1*}} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{p^x}{p^y}$$

$$\frac{p^x}{p^y} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = -\frac{\partial Y^2}{\partial X^2}\Big|_{U^2=U^{2*}}$$

The price system <u>decentralizes</u> the efficient allocation decision.

The Robinson Crusoe Model

q = oyster production
c = oyster consumption
168 (hours per week) endowment
L = labor demanded
R = leisure demanded
168-R =labor supplied

$$q = F(L) \qquad (2.1)$$

$$R = 168 - L \qquad (2.2)$$

Centralized Allocation

We assume second order conditions so that local maxima are global maxima:

$$F'' < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial R^2} < 0.$$

$$u(c,R) = u(F(L), 168 - L) \qquad (2.3)$$

$$\max_L \; u(F(L),\; 168 - L) \qquad (2.4)$$

$$\frac{d}{dL}\, u(F(L),\; 168 - L) = 0 \qquad (2.5)$$

$$u_c F' - u_R = 0 \qquad (2.6)$$

$$\left[-\frac{dq}{dR} \right]_{u = u\max} = \frac{u_R}{u_c} = F' \qquad (2.7)$$

Pareto efficient

$$MRS_{R,c} = MRT_{R,q} (= RPT_{R,q}\ )$$

Decentralized Allocation

$$\Pi = F(L) - wL = q - wL \qquad (2.8)$$

Income:

$$Y = w \cdot 168 + \Pi \qquad (2.9)$$

Budget constraint:

$$Y = wR + c \tag{2.10}$$

Equivalently, c = Y- wR = Π + wL = Π + w(168-R), a more conventional definition of a household budget constraint.

Firm profit maximization:

$$\Pi = q - wL \tag{2.11}$$

$$\frac{d\Pi}{dL} = F' - w = 0, \text{so } F'(L^0) = w \tag{2.14}$$

Household budget constraint:

$$wR + c = Y = \Pi^0 + w168 \tag{2.15}$$

Choose c, R to maximize u(c, R) subject to (2.15). The Lagrangian is

V = u(c,R) - λ (Y - wR - c)

$$\frac{\partial V}{\partial c} = \frac{\partial u}{\partial c} + \lambda = 0$$

$$\frac{\partial V}{\partial R} = \frac{\partial u}{\partial R} + \lambda w = 0$$

Dividing through, we have

$$\text{MRS}_{R,c} = \left[-\frac{dc}{dR}\right]_{u=constant} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w \tag{2.19}$$

$$wR + c = w168 + \Pi^0 \tag{2.20}$$

$$c = w(168 - R) + \Pi^0 \tag{2.21}$$

Walras' Law

Note that the Walras Law holds at all wage rates --- both in and out of equilibrium. It is not an equilibrium condition.

$Y = w \cdot 168 + \Pi = w168 + q - wL \quad = wR + c$

$0 = w(R - (168 - L)) + (c - q)$

$$0 = w(R + L - 168) + (c - q) \qquad (2.23)$$

Definition : Market equilibrium. Market equilibrium consists of a wage rate w^0 so that at $w^0, q = c$ and $L = 168 - R$, where q, L are determined by firm profit maximizing decisions and c, R are determined by household utility maximization. (in a centralized solution L=168-R by definition; in a market allocation wages and prices should adjust so that as an equilibrium condition L will be equated to 168-R).

Profit maximization at w^0 implies $w^0 = F'(L^0)$. (Recall (2.14))
Utility maximization at w^o implies

$$\frac{u_R(c^0, R^0)}{u_c(c^0, R^0)} = w^0 \qquad \text{(Recall (2.19))}$$

Market-clearing implies $R^0 = 168 - L^0, c^0 = F(L^0)$.

So combining (2.14) and (2.19), we have

$$F' = \frac{u_R}{u_c} \qquad (2.25)$$

which implies Pareto efficiency.